

Reconstruction of the $\nu = 1$ Quantum Hall Edge

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The sharp $\nu = 1$ quantum Hall edge present for hard confinement is shown to have two modes that go soft as the confining potential softens. This signals a second order transition to a reconstructed edge that is either a depolarized spin-texture edge or a polarized charge density wave edge.

Keywords: Quantum Hall edges, spin textures.

I. INTRODUCTION

In the Quantum Hall (QH) effect at ferromagnetic fillings there are excitations “skyrmions” that involve spin textures - topologically nontrivial configurations of the spins. [1] There is experimental evidence that skyrmions are the lowest energy charged excitations at $\nu = 1$. [2–4] A basic feature of these spin textures is that the topological density is proportional to the charge density. As a consequence, low energy smooth variations in the charge density can be achieved by texturing the spins. We here discuss another example where this mechanism seems to be at work, namely at the edges of QH systems. [5–7]

We consider the edge of a $\nu = 1$ QH system as the confining potential softens from a hard confinement where the edge is sharp. Calculating the excitations about the sharp edge we find two modes that soften as the confining potential softens. One is a spin flip mode and one is a polarized mode. The softening modes signal second order phase transitions to a spin texture edge and a charge density wave edge respectively. For small Zeeman energies the initial instability is to the spin textured edge. We predict that the textured edge has a new gapless mode. Experimentally, the textured edge is signalled by its sensitivity to the value of the Zeeman energy and the associated depolarization of the edge.

II. THE SHARP EDGE

Our system is a two-dimensional electron gas in a perpendicular magnetic field \mathbf{B} . The electron gas is confined to a wide bar with two straight edges. We assume the confining potential at the left edge always to be strong so that the system is a spin polarized $\nu = 1$ quantum Hall state at this edge and that this state continues deep into the bulk. The orbital Hilbert space is restricted to the lowest Landau level.

We consider the right edge of this $\nu = 1$ quantum Hall liquid as a function of the strength of the confining potential at this edge. When the potential is strong the ground state is a polarized $\nu = 1$ state where all the spin up orbitals from $k = 0$ (left edge) out to a maximum momentum k_F (right edge) are filled:

$$|\nu = 1\rangle = \prod_{0 \leq k \leq k_F} c_{k\uparrow}^\dagger |0\rangle \quad . \quad (1)$$

$c_{k\sigma}^\dagger$ creates electrons in the lowest Landau level with momentum $k = 2\pi n_k/L$, $n_k = 0, \pm 1, \pm 2, \dots$ and spin $\sigma = \uparrow, \downarrow$. The corresponding wave functions are $\varphi_k = (\sqrt{\pi}L\ell)^{-1/2} e^{iky} e^{-(x/\ell - k\ell)^2/2}$, where $\ell = \sqrt{\hbar c/eB}$ is the magnetic length. L is the length of the edge and we assume periodic boundary conditions along the edge and use Landau gauge $\mathbf{A} = Bx\hat{y}$.

The state $|\nu = 1\rangle$ has a *sharp* spin polarized edge: In momentum space the density falls discontinuously to zero, whereas in real space it falls to zero over a length of order ℓ . This is the simplest of the sharp quantum Hall edges that have been studied extensively, see e.g. [8].

The Hamiltonian is constructed by taking matrix elements of the Coulomb interaction $V(\mathbf{r}) = e^2/(\epsilon|\mathbf{r}|)$ between states in the lowest Landau level, in the presence of a background charge density $\rho_b(\mathbf{r})$ which makes the system neutral and confines the electron gas. For now, we follow standard practise and implement the confining potential by taking ρ_b to fall linearly from a constant bulk value $(2\pi\ell^2)^{-1}$ to 0 over a distance w at the edge. However, it turns out that this gives a non-generic confining potential, see below. The Hamiltonian also contains the Zeeman term $H_Z = g\mu_B B S_z$, where S_z is the component of the total spin along \mathbf{B} . The problem is characterized by two dimensionless parameters:

$\tilde{g} = g\mu_B B/(e^2/\epsilon\ell)$, the ratio of the Zeeman energy ($g\mu_B B$) to the typical Coulomb energy ($e^2/\epsilon\ell$) and $\tilde{w} = w/\ell$, which is a measure of the “softness” of the edge.

We study the edge and the edge modes as functions of \tilde{g} and \tilde{w} . When $\tilde{w} = 0$, the sharp edge ($|\nu = 1\rangle$) is the ground state for any \tilde{g} . However, when \tilde{w} increases, charge will eventually move outwards and the edge reconstructs. [9–11] The question is how this happens. It is also clear that for large enough \tilde{g} the ground state is spin polarized.

We first consider particle-hole excitations of the sharp edge $|\nu = 1\rangle$. Excitations of this ferromagnetic ground state are characterized by two quantum numbers: momentum q and spin s , corresponding to translations along the edge and rotations of the spins about the z -axis respectively. The possible ph-excitations are

$$\begin{aligned} |q, s = 0\rangle &= \sum_{k_F - q < k \leq k_F} \psi_{k\uparrow} c_{k+q\uparrow}^\dagger c_{k\uparrow} |\nu = 1\rangle \quad (\text{polarized excitations}) \\ |q, s = 1\rangle &= \sum_{0 \leq k \leq k_F} \psi_{k\downarrow} c_{k+q\downarrow}^\dagger c_{k\uparrow} |\nu = 1\rangle \quad (\text{spin flip excitations}) \end{aligned} \quad (2)$$

The wave functions $\psi_{k\sigma}$ and the energies are determined by diagonalizing the Hamiltonian in the ph-subspace.

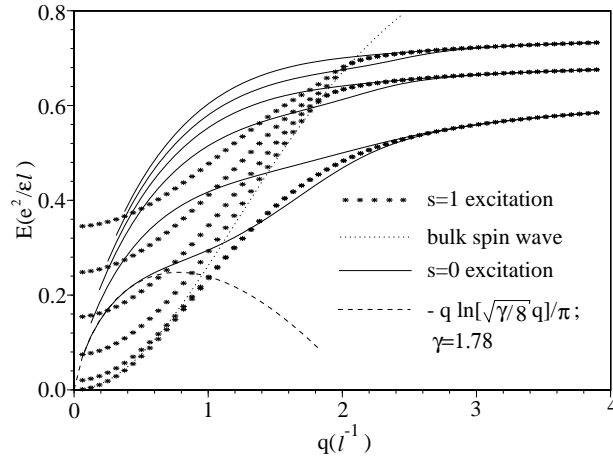


FIG. 1. Excitations of energy E and momentum q of the sharp edge $|\nu = 1\rangle$ at $\tilde{w} = 0$ and $\tilde{g} = 0$.

Fig. 1 shows the lowest energy excitations at $\tilde{w} = 0$ and $\tilde{g} = 0$ for $q \geq 0$. The $s = 0$ modes are the gapless chiral ($q \geq 0$) edge magnetoplasmons, corresponding to one-particle, two-particle etc. branches of the one dimensional massless field theory. [8] The lowest branch agrees with the analytic result $E = -\frac{q}{\pi} \ln(\sqrt{\gamma/8}q)$, $\gamma \approx 1.78$. [12] The $s = 1$ excitations are non-chiral and the gapless mode is the Goldstone mode of the quantum Hall ferromagnet. The higher energy modes all have gaps. For small q these branches are above the bulk spin wave energy, but at some critical momentum, q_c , each branch falls below this energy. For $q < q_c$, the states extend into the bulk, whereas they become localized at the edge for $q > q_c$. The Zeeman energy is included, $\tilde{g} \neq 0$, by shifting all $s = 1$ energies by \tilde{g} . (The pairing of branches seen in the figure is due to the two edges of the QH bar.)

Upon softening the confining potential, i.e., increasing \tilde{w} the lowest $s = 0$ and $s = 1$ modes soften and for each mode the energy becomes negative at some critical \tilde{w}_c (and for some q) thus signalling an instability of the sharp edge, see Fig. 2.

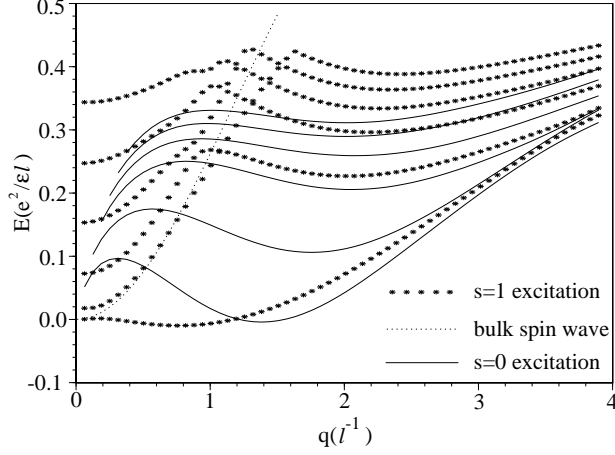


FIG. 2. Excitations of energy E and momentum q of the sharp edge $|\nu = 1\rangle$ at $\tilde{w} = 7.04$ and $\tilde{g} = 0$.

At vanishing Zeeman energy, $\tilde{g} = 0$, this happens first for the spin flip mode at $\tilde{w}_{sf} = 6.77$ and subsequently, at $\tilde{w}_{pol} = 7.29$, for the polarized mode. A nonzero \tilde{g} disfavours the spin flip mode and there is a critical \tilde{g}_c above which the spin flip instability is preempted by the polarized instability. (For the \tilde{w} -model $\tilde{g}_c = 0.016$, but for a more realistic confining potential \tilde{g}_c is likely to be larger, see below.) The polarized reconstruction where a lump of quantum Hall liquid is split off and deposited a distance ℓ from the bulk happens only at $\tilde{w}_{hole} = 9.0$. [9,10] Thus this instability is preempted by the instabilities discussed here.

III. TEXTURED EDGE AND CDW EDGE

Having identified two modes that go soft as the confinement softens thus signalling instabilities of the sharp edge, we here identify what the corresponding new ground states are. The ground state corresponding to the spin flip mode is a spin texture state

$$|\text{TEX}, q\rangle = \prod_{0 \leq k \leq k_F} (u_k c_{k\uparrow}^\dagger + v_k c_{k+q\downarrow}^\dagger) |0\rangle \quad , \quad (3)$$

where $|u_k|^2 + |v_k|^2 = 1$ (u_k, v_k can be chosen real and positive). The sharp edge is obtained if $u_k = 1$ for all k . For small deviations from the sharp edge v_k are small and $u_k \approx 1$, the ground state then becomes

$$|\text{TEX}, q\rangle \approx (1 + \sum_{0 \leq k \leq k_F} v_k c_{k+q\downarrow}^\dagger c_{k\uparrow}) |\nu = 1\rangle \quad . \quad (4)$$

The textured ground state (3) can thus be thought of as a condensation of spin flip excitations, cf. (2). The state $|\text{TEX}, q\rangle$ is a spin texture state of the same type as the one that describes skyrmion quasi particles with topological charge q . [13]

The edge spin texture can also be analyzed within the nonlinear σ -model (where the spin is represented as a unit vector $\mathbf{n}(\mathbf{r})$) that describes the long distance spin dynamics of a QH ferromagnet. [1] The edge spin texture takes the form

$$n_x + in_y = \sqrt{1 - f^2} e^{i(ky + \theta)} \quad , \quad n_z = f(x) \quad , \quad (5)$$

where $f = f(x)$ approaches 1 deep in the bulk and falls below 1 at the edge. This ansatz leads to the topological (and hence charge) density $q(\mathbf{r}) = (-k/4\pi)df/dx$. Thus the spin is polarized deep in the bulk ($n_z = f = 1$), but starts tilting as the edge is approached ($n_z = f < 1$). Moving along the edge, the spin in the edge region rotates around the z -direction with wave vector k . By numerically integrating the equations of motion obtained from the non-linear σ -model one determines $f(x)$ and k . This gives results, for $\tilde{g} \ll 1$, that agrees with Hartree-Fock calculations for the transition to the textured edge.

The ground state that corresponds to the polarized ($s = 0$) excitations in (2) is obtained by replacing \downarrow by \uparrow in (3) (this also restricts the range of k)

$$|\text{CDW}, q\rangle = \prod_{k_F - q < k \leq k_F} (u_k c_{k\uparrow}^\dagger + v_k c_{k+q\uparrow}^\dagger) |0\rangle \quad . \quad (6)$$

In this state charge is moved outwards at the price of modulating the charge density along the edge, thus forming a charge density wave (CDW) edge.

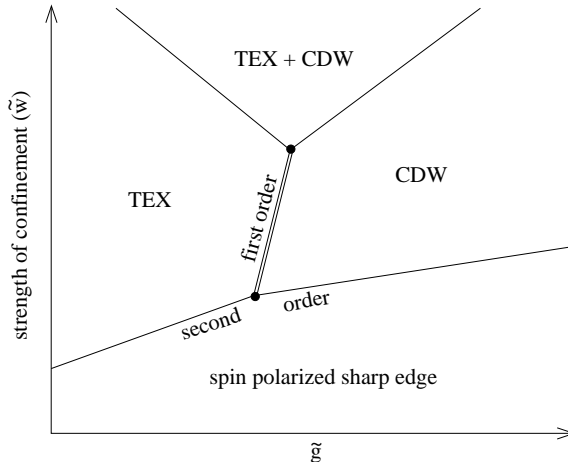


FIG. 3. Phase diagram for the $\nu = 1$ edge.

Fig. 3 shows a typical phase diagram in the $\tilde{w}\tilde{g}$ -plane. \tilde{w} should be understood as some measure of a confining potential. It turns out that only the topology of the phase diagram is stable under changes in the confining potential, whereas the position of the phase boundaries (and hence \tilde{g}_c) is very sensitive. In particular, one finds that the slope of the softening $s = 1$ dispersion curve vanishes at zero momentum for the \tilde{w} -model. Generically, the slope will be negative when the potential softens and this will favour the spin flip instability (increasing \tilde{g}_c) since the $s = 1$ mode goes as q^2 whereas the $s = 0$ mode goes as $q \ln q$. It is also possible to have a combination of a spin texture and a charge density wave edge as indicated in Fig. 3. [7]

IV. EXCITATIONS OF THE TEXTURED EDGE

The sharp edge, $|\nu = 1\rangle$, is invariant under translations along the edge, t_y , as well as under rotations of the spins around the z -axis, s_z . The textured edge, in Hartree-Fock (3) or in the effective theory (5), is invariant only under the linear combination $t_y + qs_z$, whereas the orthogonal combination is spontaneously broken. (In (5), the angle θ labels the degenerate ground states and the degenerate microscopic states are obtained from (3) by making a rotation of the spins around the z -axis.) As a consequence of the broken symmetry there is a gapless Goldstone mode. Note that the broken symmetry is a symmetry also in the presence of the Zeeman term, thus the mode is gapless also for $\tilde{g} \neq 0$. When quantum fluctuations are included the broken symmetry will be restored. However, we expect an algebraic long range order and that the gapless mode survives.

By first determining the textured ground state, at some point (\tilde{w}, \tilde{g}) , in Hartree-Fock and then considering ph-excitations, corresponding to (2), we obtain the excitations of the textured edge.

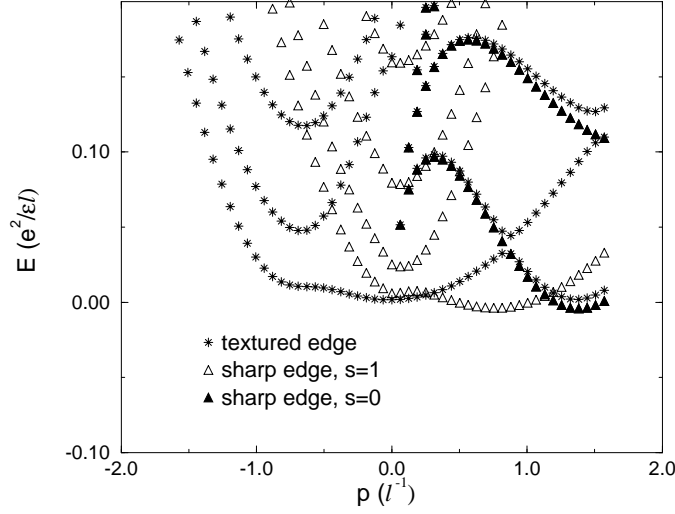


FIG. 4. Excitations of energy E and momentum p of the textured edge at $\tilde{w} = 7.0$, $\tilde{g} = 0.006$ and $q = 0.75$.

Fig. 4 shows the result for $\tilde{w} = 7.04$, $\tilde{g} = 0.006$ and $q = 0.75$. This is close to the transition from the sharp edge and the modes for the sharp edge have been included for comparison. (The data for the sharp edge is also for $\tilde{w} = 7.04$, $\tilde{g} = 0.006$; it is not the ground state at this point.) We see that the polarized modes evolve smoothly into new modes of the textured edge, whereas the lowest spin flip mode becomes gapless and has a very flat dispersion relation. This mode is concentrated at the edge of the system where $v_k \neq 0$. The higher spin flip modes have their minimum translated to $p = -0.75$.

V. DISCUSSION

It is believed that standard quantum Hall samples may contain highly reconstructed edges with compressible and incompressible regions even for the integer quantum Hall states. [14] If this is correct then to see the edge reconstructions to a textured (or a charge density wave) edge it may be necessary to have a sharper edge, possibly produced by cleaving. [15]

The main experimental signature of the textured edge is likely to be that it is depolarized and that it depends strongly on the Zeeman energy. It could be investigated by tunneling into the edge at various values of a tilted magnetic field or by using NMR.

The edge reconstructions discussed here for the $\nu = 1$ edge may take place also at other ferromagnetic filling factors as well as in quantum dots. [16,17]

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